Infinite Series Tests

Test	Series	Convergence Condition(s)	Divergence Condition(s)	Notes
Group 1 – First Test You Should Always Try				
Divergence (nth-Term)	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test can be used only for divergence.
Group 2 –Special Series				
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	Sum: $S = \frac{a}{1-r}$; $\lvert r \rvert > 0$
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	1: $\lim_{n\to\infty} a_n = 0$ and 2: $0 < a_{n+1} \leq a_n$		Condition 1 is the Divergence Test; Remainder: $\lvert R_N \rvert \leq a_{N+1}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		L is finite; Sum: $S = b_1 - L$
Group 3 – Core Series Tests (In Order of Use)				
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
Limit Comparison $(a_n, t_n > 0)$	$\sum_{n=1}^{\infty} a_n$	$0 \leq \lim_{n\to\infty} \frac{a_n}{t_n} < \infty$ and $\sum_{n=1}^{\infty} t_n$ converges	$\lim_{n\to\infty} \frac{a_n}{t_n} > 0$ and $\sum_{n=1}^{\infty} t_n$ diverges	$\sum_{n=1}^{\infty} t_n$ is the test series.
Direct Comparison $(a_n, t_n > 0)$	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq t_n$ and $\sum_{n=1}^{\infty} t_n$ converges	$0 < t_n \leq a_n$ and $\sum_{n=1}^{\infty} t_n$ diverges	$\sum_{n=1}^{\infty} t_n$ is the test series.
Integral (f is continuous, positive and decreasing for x>k)	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_k^{\infty} f(x)\,dx$ converges	$\int_k^{\infty} f(x)\,dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)\,dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} > 1$	Use when you have something like $[f(n)]^n$; Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = 1$